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ENERGY LOSS AND STRAGGLING OF PROTONS AND DEUTERONS

BY

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Synopsis

The energy loss and straggling of protons and deuterons have been measured in the energy range from 1.5 to 4.5 MeV. The elements investigated are Be, Al, Ni, Cu, Ag, and Au. The results are plotted in such units that all the points are expected to fall on a single curve. This is found to be the case and the shape of the curve is in good agreement with theoretical expectations.

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I. Introduction

The present experimental investigation is concerned with the energy loss and straggling which protons and deuterons suffer when they penetrate foils of various elements.

The particles were accelerated in the 4.5 MV electrostatic accelerator at the Institute for Theoretical Physics in Copenhagen. A magnetic spectrograph was used as a precision instrument for the energy determinations.

In the energy range below 2 MeV numerous stopping power investigations have been carried out previously, but in the range from 2 to 10 MeV the experimental data are rather scarce. At higher energies ($\gtrsim$ 10 MeV) many investigations have been performed by means of cyclotrons. However, in general, the latter measurements are made only at the fixed energy which the cyclotron in question yields, and consist in a determination of the stopping power of various elements relative to a given standard. Air or aluminum are often used as such standards[6, 13]; this is unfortunate since, in particular for air, the results obtained from different experiments vary considerably. For this reason, it is difficult to combine the various results to an accurate description of the stopping phenomenon.

The present measurements are performed on metal foils of beryllium, aluminum, nickel, copper, silver, and gold, and the investigations cover the energy range from 1.5 to 4.5 MeV.

II. Summary of Theory

The penetration of charged particles through matter has been studied theoretically by many authors. The topic has been surveyed by NIELS BOHR[4] who, in particular, has discussed the conditions under which the various approximations to the problem can be applied.

The slowing down of a proton or a deuteron is caused by electronic collisions in which the energy is transferred to individual electrons in the atoms, resulting in atomic excitation and ionization processes. In a nuclear collision, the momentum is taken up by the target atom as a whole and,

because of the much larger mass involved, such collisions do not contribute essentially to the energy loss. They do, however, give rise to the multiple scattering which the particle suffers by the penetration, whereas the electronic collisions are of minor importance for this effect.

The energy loss. For the bombarding energies employed in the present investigation, the electronic collisions correspond to small values of the collision index $\varkappa$, i. e.,

$$\varkappa = 2\frac{Z_1 e^2}{\hbar v} < 1\,, \tag{1}$$

where $Z_1 e$ and v are the charge and velocity, respectively, of the bombarding particles. Under such circumstances the Born approximation can be employed, and on this basis BETHE[1] has shown that the average energy loss dE per range interval dR for non-relativistic particles is given by

$$\left.\begin{aligned} \frac{dE}{dR} &= \frac{4\pi e^4 {Z_1}^2}{mv^2} NZ_2 L\,, \text{ where} \\ L &= \log_e\left\{\frac{2mv^2}{I}\right\} \end{aligned}\right\} \tag{2}$$

In these expressions, e and m are the charge and mass of an electron, whereas N represents the density of the atoms in the target material which has the atomic number Z_2.

The energy I is an average over the excitation and ionization energies for the electrons in the target atoms. The average involves only those electrons which contribute to the stopping, i. e., electrons which have orbital velocities smaller than or comparable to the projectile velocity v. This implies that I will depend on the projectile energy E, unless

$$E \rangle\rangle \frac{A_1 M_0}{m} I_s \text{ for all } s, \tag{3}$$

where $A_1 M_0$ is the mass of the projectile, M_0 the nuclear mass unit, and I_s are the ionization energies of the various electrons in the atoms.

For very high (but non-relativistic) energies, where these conditions are all fulfilled, BLOCH[3] has shown that employment of the Thomas-Fermi statistical model of the atom leads to an average excitation potential

$$I = Z_2 I_0\,, \tag{4}$$

where I_0 is a constant of the order of the Rydberg energy $R_y = 13.6$ eV.

For the K-electrons the inequality (3) requires that even when bombarding an element as light as aluminum, the proton energy should be larger than 4 MeV. This means that, in the present investigation, the K-electrons of the heavier elements do not yield any significant contribution to the stopping, and that for aluminum one has to apply a correction to the simple expression (4) corresponding to a velocity dependence given by

$$I = Z_2 I_0 \exp\left\{\frac{C_K}{Z_2}\right\}, \tag{5}$$

where I_0 is the constant found at higher energies. BETHE and WALSKE[2,14] have calculated this C_K-correction and they find for low bombarding energies that C_K is negative, whereas for energies in the transition region (e. i., corresponding to an equality sign in (3) for the K-shell) it passes through a positive maximum before it goes to zero when (3) becomes valid.

For lower energies or heavier elements similar corrections would have to be applied also to other shells, but such calculations are only available for the L-shell[15]. However, it has been shown by LINDHARD and SCHARFF[11,12] that, to the extent the Thomas-Fermi model can be applied, one should expect the function L to be dependent on v and Z_2 in such a manner that

$$L = L\{x\}, \text{ where } x = \frac{v^2}{v_0^2 Z_2} \tag{6}$$

and

$$v_0 = e^2/\hbar .$$

It is evident that this is true for Bloch's solution for large values of x, where

$$L = \log_e\left\{\frac{2mv^2}{Z_2 I_0}\right\} = \log_e\left\{\frac{4R_y}{I_0}\right\} + \log_e\{x\}, \tag{7}$$

but (6) holds also in the general case where the function L has not been calculated explicitly.

The result that L to a first approximation depends only on a single parameter is of great value, as it makes it possible to present the experimental data in a comprehensive form which is well suited for interpolations to other elements than those actually measured.

The energy straggling. As a consequence of the statistical nature of the collision processes, not all the particles in a mono-energetic beam will lose the same amount of energy when they penetrate a foil of a definite thickness ΔR.

The standard deviation Ω of the energy distribution is called the energy straggling. The main contribution to Ω comes from the more violent collisions by which the electrons are given kinetic energies close to the maximum energy $4\,mv^2$ which a free electron can obtain in a collision with the projectile. This energy is large compared to I_s when the inequalities (3) are satisfied, and under these conditions one finds (ref. 4))

$$\Omega^2 = Z_1^{\,2} \cdot 4\pi e^4 Z_2 N \Delta R. \tag{8}$$

For lower energies where (3) is no longer valid, LINDHARD and SCHARFF[11] have shown that Ω can be expressed in terms of the function L which, according to equation (2), determines the average energy loss. They find that

$$\Omega^2 = Z_1^{\,2} 4\pi e^4 Z_2 N \Delta R \cdot \frac{1}{2} L\{x\} \tag{9}$$

$$\text{for } L\{x\} \lesssim 1.$$

A smooth transition between the two approximations is expected to take place for $L\{x\} \simeq 2$, but a more precise estimate is not available in this region.

III. Experimental Procedure

A thin layer of gold deposited on a carbon foil was bombarded with, e. g., the proton beam. The protons scattered at a backward angle of 145 degrees were passed through a broad-range magnetic spectrograph[8] and recorded by means of a photographic emulsion placed along the focal plane. The plate was exposed twice, first with a foil inserted in the primary beam and then, immediately afterwards, with the foil removed. As an example, Fig. 1 shows the corresponding groups of protons scattered from the gold layer. Protons scattered from the carbon backing have much lower energies at backward scattering angles, and do not disturb the measurements. The narrow peak obtained without the stopping foil shows that the energy spread of the primary beam is less than ± 0.1 per cent. The peak obtained with the foil inserted in the beam is shifted towards a lower energy and has an increased width. These effects are caused by energy loss and straggling in the stopping foil.

The two exposures corresponded to the same number of primary protons, as measured by collecting the beam in a Faraday chamber behind the carbon foil and recording the accumulated charge by means of a beam

integrator. The total number of tracks in each of the two peaks should thus be very nearly equal. It is determined by the thickness of gold scatterer and the bombarding conditions. In most of the exposures one aimed at

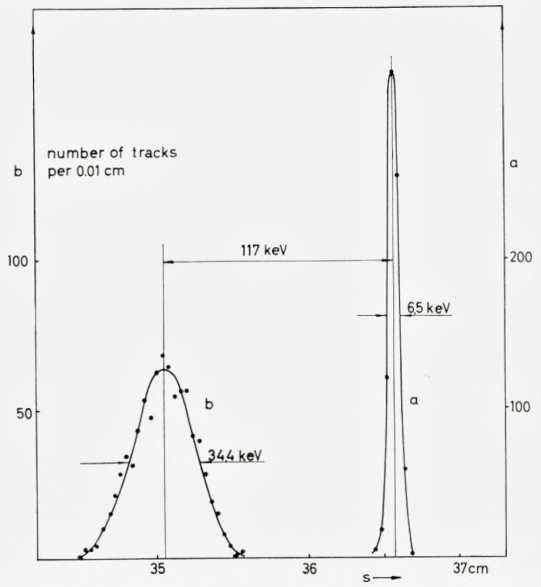

Fig. 1. Spectrum of protons scattered from a thin gold target, a) without foil and b) with a 3.3 mg/cm² Au-foil inserted in the 3 MeV proton beam. The spectrum was obtained by means of a magnetic spectrograph and the particles were recorded in a photographic emulsion.

about 2000 tracks, and this number could be obtained in a few minutes by collecting a charge of the order of 10 micro-coulomb in the Faraday chamber.

Determination of the energy loss and straggling. The spectrograph was calibrated by recording α-particles from $_{84}Po^{210}$ [5] at various field strengths, as measured by a nuclear induction Gauss-meter. By means of the calibration curve the average energy can be determined for each of the two peaks in Fig. 1, and from these energies the mean energy of the particles in the investigated foil can be computed as well as the average energy loss. This involves a correction for the recoil energy lost in the gold scatterer (1.8 per cent for protons scattered through 145 degrees) and, strictly speaking, one should also take into account that the energy scale is non-linear; however, the widths of the peaks are so small that this effect is quite negligible.

In a similar way, the energy straggling may be obtained from the standard deviations of the two distributions. The width of the broad curve is due mainly to the straggling phenomenon and the distribution may be expected to be approximately Gaussian. By plotting the integral distribution, i. e., the area $H\{s\}$ indicated in Fig. 2, one should therefore obtain an S-shaped curve with a steepest slope proportional to the reciprocal of the

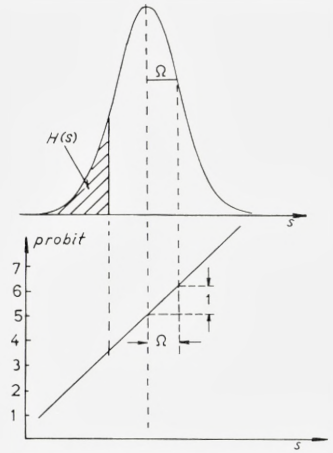

Fig. 2. Determination of the standard deviation of a Gaussian distribution by means of 'probits' (cf. section III).

standard deviation. In order to exploit all the points for the determination of this slope, the curve can be transformed to a straight line with the same slope by plotting the so-called probits (cf. Fig. 2) which represent a linear transform of the integrated Gaussian (cf., e. g., ref. 9).

From the standard deviation Ω_2 determined in this way for the energy distribution corresponding to the broad peak, one finds the energy straggling Ω itself by subtracting the contributions from other effects. The measured distribution results from a folding of the straggling curve with the curve which would be obtained if the straggling phenomenon was absent. Fortunately, the standard deviations add up geometrically, i. e., if we denote the standard deviation of the latter curve by Ω_0, then

$$\Omega^2 = \Omega_2^2 - \Omega_0^2 . \tag{10}$$

Hence, the corrections have a relatively small influence and for this reason the exact magnitude of Ω_0 is not important. One contribution to Ω_0 comes from the finite resolution of the spectrograph and the analyzing

magnet of the accelerator. It is given by the standard deviation Ω_1 of the reference curve ('a' in Fig. 1) and it can, for the present purpose, be estimated sufficiently well from the directly measured half-width.

Another contribution to Ω_0 comes from the multiple scattering in the stopping foil which smears out the beam spot on the target because of the spacing D needed between foil and target (D = 1.8 cm, cf. Fig. 3). The

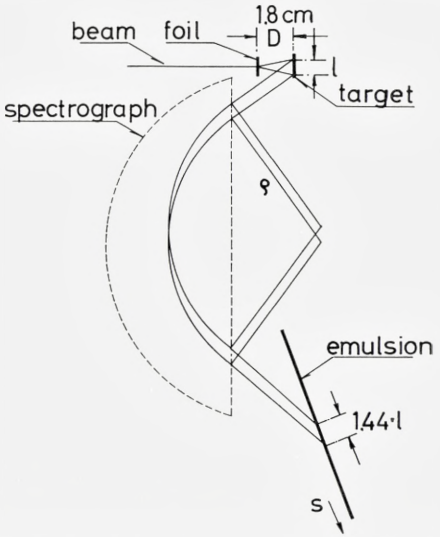

Fig. 3. Sketch of the spectrograph (not to scale), showing the broadening effect caused by multiple scattering in the stopping foil. The size of the effect is determined by the distance 'D' between foil and target and it is further magnified 1.44 times by the spectrograph. Two typical orbits are indicated.

magnitude Ω_s of the contribution from this effect was not measured directly, but it can be estimated from earlier measurements (cf., e. g., ref. 10) and from theory by the following considerations.

The standard deviation σ for the projected angular distribution is given by [4]

$$\sigma = \frac{Z_1 Z_2}{\sqrt{2}} \cdot \frac{[\pi e^4 \; N \; \Delta R \; \log_e \{\bar{n}\}]^{1/2}}{E}, \tag{11}$$

where $\bar{n}$ is the average number of nuclear collisions which a particle encounters by the passage of the foil. Hence Ω_s can be calculated from the expression

$$\Omega_s = \frac{dE}{ds} \cdot F \cdot D \cdot \sigma, \tag{12}$$

where $F = 1.44$ is the (constant) magnification and

$$\frac{dE}{ds} = 0.87 \frac{E}{\varrho} \tag{13}$$

is the reciprocal energy dispersion of the spectrograph.

Since for most of the exposures the magnetic field was adjusted to give a radius of curvature $\varrho \simeq 30$ cm, it is found from equations (11), (12), (13), and (8) that

$$\frac{\Omega_s}{\Omega} \simeq 2.7 \cdot 10^{-2} \sqrt{Z_2 \log_e \{\bar{n}\}}, \tag{14}$$

if $L\{x\} \gtrsim 1$. Hence, the correction factor is nearly independent of ΔR and E which enter only through the logarithmic term under the square root. In accordance with the theory it is found experimentally[10] that, for foil thicknesses of the order of mg/cm^2 and energies of the order of MeV, the square root decreases from a value of 15 for gold to 12 for copper, whereas for aluminum it is expected to be as low as 9. Because of the geometrical addition of the standard deviations, these values imply that even in the case of gold the correction to Ω_2 amounts only to approximately 8 per cent.

In order to ensure that the actual Ω_s correction was not underestimated, a few exposures were made with a target where the thin gold layer was confined to a narrow line, only 1 mm wide. A scatterer of this shape acts as a line source in the spectrograph even when a foil is introduced in the beam, and hence in this case no Ω_s correction is needed (cf. section IV). The energy straggling for the observed particles should be the same in the two geometries since the stopping and scattering in the foil is caused by two different processes (cf. section II) and therefore not correlated.

In addition to the above mentioned contributions to the widths of the observed peaks, one has to consider the effect of inhomogeneities in the investigated foils, and the degree of homogeneity of course depends on the technique by which the foils have been produced.

Preparation of the foils and determination of their thicknesses. The main source of error in stopping experiments comes from the difficulties involved in producing clean and very homogeneous foils. Great care is

needed in the preparation, and it is essential to check the homogeneity and cleanness of the foils sufficiently well to make the experimentally determined weight per unit area an accurate measure of ΔR.

The foils were weighed on a balance which could be read with an accuracy better than $\pm 10\ \mu g$. The absolute calibration of the balance was checked against a standard weight. The lightest foil weighed about 3 mg and the relative accuracy of the weighings was therefore better than 1 per cent.

The areas were computed from the linear dimensions of the foils which for the larger ones were measured by means of a movable microscope table calibrated to better than 10^{-2} mm. The measured areas were about 1 cm^2 or larger, and the relative uncertainty was less than 1 per cent.

The beryllium foils were not made with the present experiment in mind. They were rather thin and it was therefore necessary to use several layers together. The foils had a somewhat irregular shape, and for this reason the areas were determined by making blue prints on a homogeneous piece of paper. The figures were cut out and weighed relative to a piece of known area.

The aluminum foils were very uniform, rolled foils with a stated purity of 99.6 per cent. The impurities were mainly iron and silicon and the correction to the stopping power was therefore negligible. The nickel foils and some of the copper foils were rolled foils, produced commercially. The stated purity was better than needed for the present experiments.

The remaining copper foils, and all foils of silver or gold were made by evaporation in vacuum from a heated tungsten ribbon. The metal was evaporated onto a glass plate which had been prepared in the following way. First it was cleaned in sodium hydroxide and then a solution of polystyrene in chloroform was poured over it. After the chloroform had evaporated, the plate was left with a thin coating of polystyrene which was used as a basis for the evaporated metal. By means of a razor blade, the metal foil was then cut into rectangular pieces, each approximately 1 cm^2, which is the size of the standard frames used. Because of the polystyrene film, the foils came off the glass quite easily when a drop of water was added. Subsequently, the polystyrene was removed by dissolving it in chloroform. The areas of the rectangular pieces were measured both before they were taken off the glass and after the final preparation. They showed no tendency to shrink if the cuts were not made before the foil had reached room temperature after the evaporation.

The purity of the evaporated foils was checked by employing the same material for production of a very thin evaporated target from which protons could be scattered elastically and measured in the spectrograph. Each

contaminant gives rise to an elastic peak in the spectrum, and from the height of these peaks very small relative concentrations of the impurities can easily be determined. It was found that the gold did not contain any impurity large enough to justify a correction. Some of the silver foils contained 1.3 per cent of copper and this implies a correction to the energy loss of approximately 0.3 per cent, which is somewhat less than the estimated uncertainty of the measurements. The correction was included because the deviation was one-sided.

The foils were selected with respect to homogeneity by weighing neighbouring foils from the same evaporation, and if the deviations in thickness exceeded 2 per cent, the foils were not used. In order to reveal more localized inhomogeneities of the foils, investigations were made by means of a small range-meter, similar in construction to that described by CHILTON et al.[7]. A thin Po-source was placed on a movable table below a diaphragm with a small hole, comparable in size to the 1 mm^2 beam spot of the accelerator. The foil to be investigated was placed on the diaphragm. Some of the α-particles emitted from the source passed through the hole, penetrated the foil, and entered a Geiger counter. A plot of the counting rate versus the vertical position of the table yielded a well-defined half-intensity point corresponding to the range of the particles. In this position variations in the thickness of the foil were directly indicated by a change in counting rate when the foil was moved with respect to the diaphragm so that different parts of the foil were exposed.

The method was very sensitive and the foils were discarded if the measured thickness fluctuations exceeded 1.5 per cent. From the measurements one could extrapolate to the most probable value for the thickness at the center of the foil where the accelerator beam passed through in the actual experiment. The range-meter could not be used for some of the thicker foils because the range of the Po α-particles was too short. The homogeneity of these foils had to be checked in a more laborious way by bombarding them with the accelerator beam penetrating in several different positions. If the energy loss varied more than 2 per cent, the foils were discarded.

IV. Results and Discussion

The specific stopping power. In Figs. 4 and 5 the measured energy losses in KeV per mg/cm^2 are plotted as a function of the energy in MeV of the protons and deuterons, respectively. Measurements were made on 4 to 5 different foils of each of the metals indicated, the thicknesses ranging from

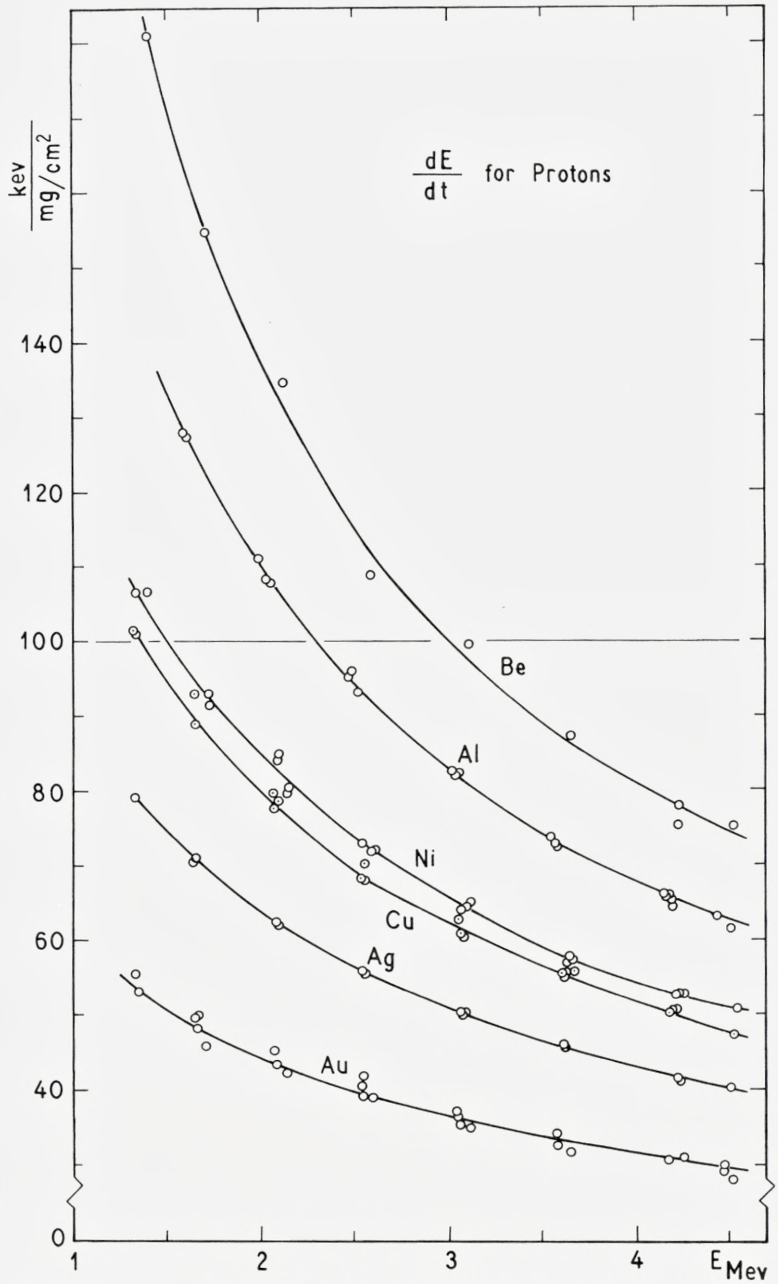

Fig. 4. Stopping powers for protons. The curves are experimental and drawn only in order to facilitate energy interpolations.

0.5 to 3.5 mg/cm^2. The uncertainty of the individual measurement is caused mainly by foil inhomogeneity, and is in the range from 1 to 3 per cent. The most reliable results were obtained for Al and Ag, the corresponding foils being particularly homogeneous. The measurements on Be were made only once, because the few foils available broke when they were taken out after the first exposure; the majority of the other measurements were repeated.

Figs. 4 and 5 are not well suited for interpolating the measured stopping powers to other elements. As mentioned in section II, it is more convenient to plot the data as a function of the parameter

$$x = \frac{v^2}{v_0^2 Z_2} = \frac{40}{A_1 Z_2} E_{\mathrm{MeV}}, \tag{15}$$

where E_{MeV} is the bombarding energy in MeV. As the measured thicknesses are given in units mg/cm^2, it is advantageous to introduce

$$dt = A_2 M_0 N \, dR \tag{16}$$

in the theoretical formulae. Equation (2) may then be rewritten in the form

$$\frac{dE}{dt} \sqrt{E Z_2} \left[\frac{A_2}{2 Z_2} \frac{1}{Z_1^2 \sqrt{A_1}} \right] = C \frac{L\{x\}}{\sqrt{x}}, \tag{17}$$

where

$$C = \frac{\pi \sqrt{2}\, e^2 \hbar}{m \sqrt{M_0}} = 14400 \frac{(\mathrm{KeV})^{3/2}}{\mathrm{mg/cm^2}}. \tag{18}$$

A plot of the experimental values of the quantity on the left-hand side of equation (17) as a function of x given by equation (15) should therefore give points falling on a single curve for all elements and projectiles. Since the measured dE/dt values are roughly proportional to $(E Z_2)^{-1/2}$, such a plot allows all the points to be presented with comparable precision, as shown in Fig. 6.

Within the experimental uncertainty, there are no differences between corresponding proton and deuteron points for a given element. This is not surprising since the mass is of negligible importance when the projectiles are very much heavier than the electrons. The points for different elements also fit in rather well with each other, although for the heavy elements there are deviations of a magnitude comparable to the experimental uncertainty. It is not clear whether the deviations are significant or not; on

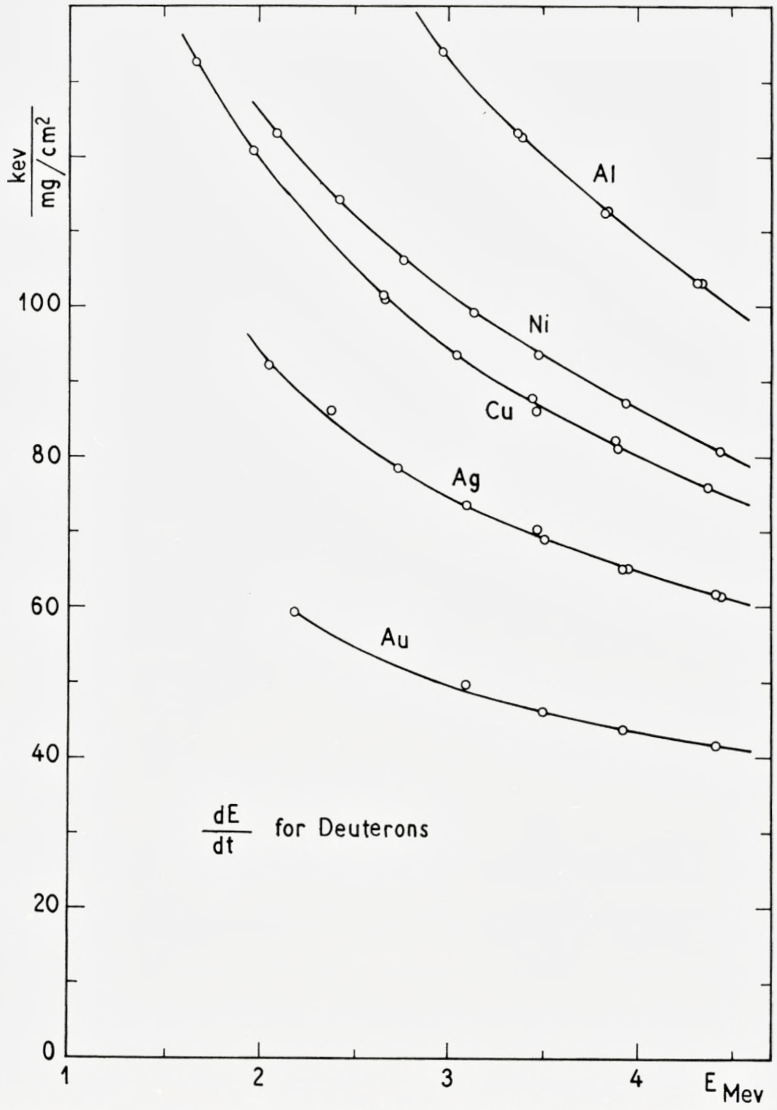

Fig. 5. Stopping powers for deuterons. The curves are experimental and drawn only in order to facilitate energy interpolations.

the other hand it would not be surprising if minor deviations occurred, since the Z_2 dependence is inferred from more specific assumptions. However, relatively safe interpolations to the stopping power of other elements

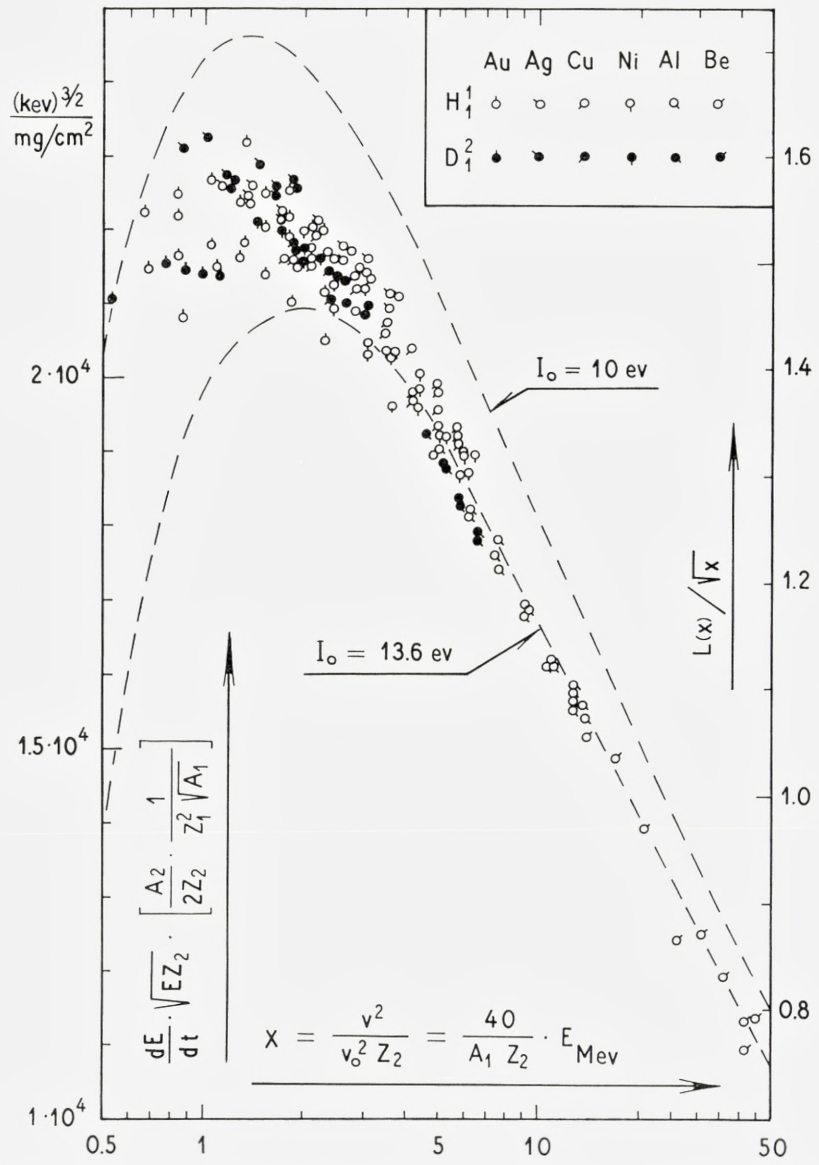

Fig. 6. Plot of the stopping power data contained in Figs. 4 and 5. According to theory, all points should fall on a single curve (cf. section IV). The factor in the square bracket does not depend much on the target material and is close to 1 for protons. The theoretical curves represent equations (2), (4), and (16), corresponding to the I_0-values indicated.

can be made from Fig. 6 in the investigated range of x-values. WARD WHALING[16] has compiled most of the existing data from stopping power measurements in the energy range below 2.5 MeV, as well as the data for gold investigated previously in the range from 1.5 MeV to 5 MeV; on this basis, he has extrapolated the results up to 10 MeV by means of the theoretical formulae. The present results agree well with the curves given by WHALING. In a few cases, the new points indicate slightly lower values, but the differences are less than 3 per cent.

In Fig. 6 the experimental data are compared with the theoretical expressions (4) and (2), corresponding to I_0-values of 10 eV and 13.6 eV. As explained in section II, the average excitation potential must at low bombarding energies be smaller than the constant value (4) which is approached at higher energies. From the figure it is evident that this is true, and the effect is shown more quantitatively in Table A, where the magnitudes of I have been evaluated at the various energies by means of equation (2) and the experimental curves in Fig. 4.

TABLE A. The average excitation potential I as derived from a comparison of formulae (2) and (16) with the experimental stopping power curves in Fig. 4. The proton energy is denoted by E_p, and ΔI represents the estimated uncertainty on I. The last row gives the ratio between the value of I at $E_p = 4$ MeV, and the atomic number Z_2 of the element in question (cf. eq. (4))

E_p	I_{Be}	I_{Al}	I_{Ni}	I_{Cu}	I_{Ag}	I_{Au}
MeV	eV	eV	eV	eV	eV	eV
1.5		185	365	377	556	866
2.0	56	184	368	371	572	937
2.5		180	371	379	576	974
3.0	56	180	371	382	588	995
3.5		175	378	378	587	1010
4.0	56	175	373	371	583	1000
ΔI	± 4	± 3	± 8	± 8	± 7	± 20
$I\{4\}/Z_2$	14.0	13.5	13.2	12.8	12.4	12.7

It is interesting to note that the variation of I for Al seems to go in the opposite direction. This is in agreement with the fact that, for high energies, various investigators[13] have found relatively low values, viz. $I_{Al} \simeq 163$ eV. The sign of the effect may be understood (cf. section II) by considering

that the C_K-correction in the present range of bombarding energies obtains its (positive) maximum value for elements in the neighbourhood of Al, as shown in Table B. From the table it appears, however, that the calculated C_K-correction does not suffice to account for the variation of the I_{Al}-values displayed in Table A.

TABLE B. Effect of the C_K-correction for Al. The correction has been estimated from the curve given by WALSKE[14].

E_p	C_K	I_{Al}^{uncorr}	I_{Al}^{corr}
MeV		eV	eV
1.5	0.3	185	181
2.0	0.6	184	174
3.0	0.9	180	168
4.0	1.0	175	162

The energy straggling. If the foil thickness t is introduced in accordance with equation (16), the relations (8) and (9) for the energy straggling Ω can be written in the form

$$\frac{\Omega}{\sqrt{t}}\sqrt{\frac{A_2}{2Z_2}\frac{1}{Z_1}} = \sqrt{\frac{2\pi e^4}{M_0}} \times \begin{cases} 1 \text{ for} & L \gtrsim 2 \\ \sqrt{\frac{1}{2} L\{x\}} \text{ for} & L \lesssim 2 \end{cases} \tag{19}$$

where

$$\sqrt{\frac{2\pi e^4}{M_0}} = 8.85 \frac{KeV}{(\text{mg/cm}^2)^{1/2}}. \tag{20}$$

In Fig. 7 the experimental values of the quantity on the left-hand side of equation (19) are plotted as a function of x. However, the plot does not include the Ω_s-correction. The magnitude of this correction can be estimated by means of the equations (11) to (14), and for each element the corresponding ordinate correction is indicated in the figure by the length of the arrows. The few exposures made with the 'line' target described in section III are consistent with the corrected values, but the points scatter too much to allow a quantitative determination of Ω_s. With the Ω_s-corrections included, the data conform quite well with theoretical expectations represented by the two curves. They are drawn in accordance with equations (19) and

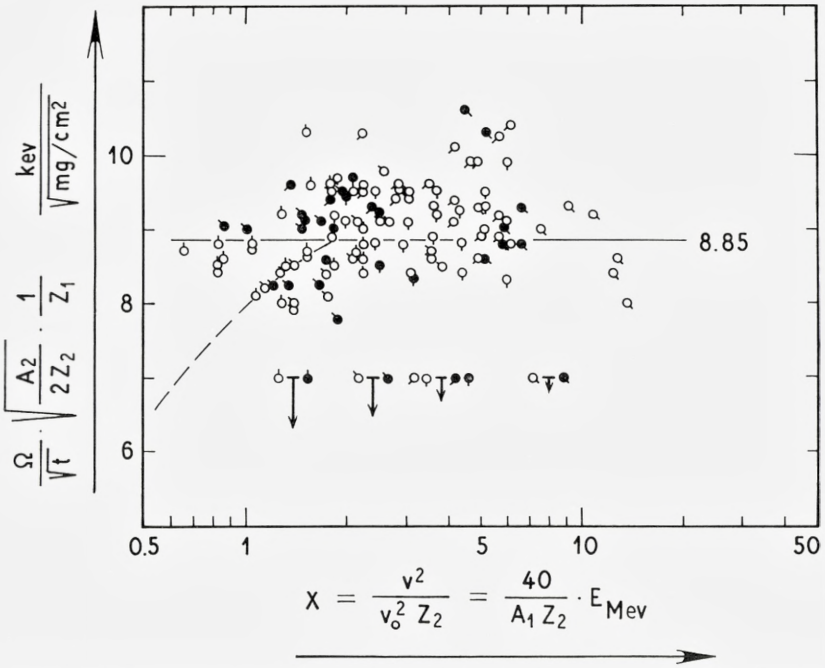

Fig. 7. Plot of the straggling data. The symbols are the same as in Fig. 6. The points have not been corrected for the scattering effect discussed in section III, but the magnitude of these corrections is indicated for each element by the length of the arrows. The curves represent the theoretical expressions (19), with the values of $L\{x\}$ derived from the experimental points in Fig. 6.

(20), and with $L\{x\}$ derived from the experimental points in Fig. 6, i. e., from the measured stopping powers.

Evidently, the points in Fig. 7 scatter too much to establish with certainty the expected decrease for small x-values, but it is difficult to obtain more reproducible data because of miscroscopic inhomogeneities in the foils. One variety of the commercial Al-foils, e. g., yielded rather large Ω-values, but these foils were only shiny on one of the sides and could therefore be discarded as far as the straggling measurements were concerned. Also the Be-foils were too poor to justify an evaluation of the straggling from the data obtained; Be is therefore not included in Fig. 7.

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